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A DISCONTINUITY STUDY IN SHIELDED COPLANAR LINE

by

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The impedance (Z_o) has been calculated by using a variational method. The propagation constant ($\beta(\omega)$) and effective dielectric constant (ε_{reff}) of the SCPL have been calculated by using the method of moments. The cut-off frequency of this waveguide has been obtained by a theorem of Van Bladel and Higgins.

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A Discontinuity Study in Shielded Coplanar Line

by

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Submitted in partial fulfillment of the
requirements for the degree of

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ABSTRACT

A study of the shielded coplanar line (SCPL) is presented. The main goal of this thesis is to develop an equivalent circuit model for a typical discontinuity in the SCPL. The formulation is based on Galerkin's method, using Green's function applied in the Fourier transform domain.

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I. INTRODUCTION

The shielded coplanar line (SCPL) is useful for radar and communication circuits at microwave and millimeter-wave frequencies. In order to use this transmission medium in the construction of microwave circuits and filters, it is necessary to have valid circuit models for typical discontinuities such as the series gap in line, open-ended stub, and a discontinuous change in width. There is a definite need for an accurate full-wave analysis of strip transmission line structures. By full wave analysis is implied the process of rigorously solving the frequency dependent electromagnetic (EM) boundary value problem with retention of all the field components. In this thesis, the boundary value problem associated with the discontinuity structure of interest incorporated in a SCPL resonator has been solved in a formulation employing full-wave analysis. The solution of the problem has been derived using an efficient method. Specifically, the derivation of the characteristic equation for resonant frequencies of a resonator model is carried out using Galerkin's technique applied in the spectral or Fourier transform domain instead of the space domain. The resonant frequency of the structure of interest is obtained

by numerically solving the characteristic equation. The details of the analysis method will appear in Chapter II of this work.

In Chapter III, at the discontinuity in an open-ended resonator, the fringing capacitance of the open end will be calculated by two methods.

II. THEORETICAL ANALYSIS

The shielded coplanar line (SCPL) to be analyzed is shown in Fig.1. A strip conductor is located symmetrically between ground strips on both sides. The strips are placed on the dielectric substrate. The SCPL is constructed by placing dielectric and conductor within a closed channel forming a shield. Dimensions must be sufficiently small to avoid propagation of waveguide modes within the channel.

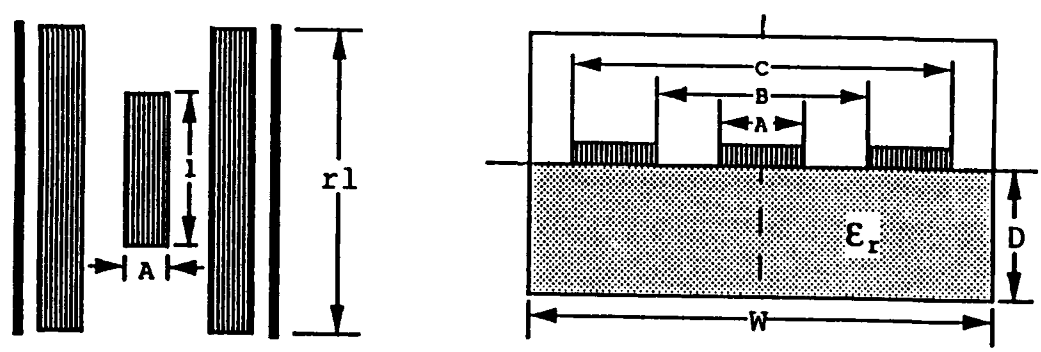


Figure 1. Top and End view of SCPL

It is assumed that the thickness of the conducting strip is negligible and that all the media and conductors are lossless. For simplicity, the

center and the ground strip are to be located symmetrically in the z-direction within the ends of the shielding enclosure.

A. FOURIER-TRANSFORMED GALERKIN'S METHOD FOR THE SCPL RESONATOR

The analysis of wave propagation on the SCPL systems of the type considered here has been carried out by Itoh and Uwano [Ref.2]. These authors treated the problem using Galerkin's method of moments in the Fourier-Transform domain. Their work is summarized in terms of a set of Green's function equations:

$$\tilde{Z}_{zz}\tilde{J}_z + \tilde{Z}_{zx}\tilde{J}_x = \tilde{E}_z \tag{1a}$$

$$\tilde{Z}_{xz}\tilde{J}_z + \tilde{Z}_{xx}\tilde{J}_x = \tilde{E}_x \tag{1b}$$

Where $\tilde{J}_z = \tilde{J}_z(k_n, \beta_m)$, $\tilde{J}_x = \tilde{J}_x(k_n, \beta_m)$.

In Eqs.(1a) and (1b), J_z and J_x are the z and x components of the currents on the strip conductor, and E_z and E_x are the components of electric field tangent to the substrate surface, and the $\tilde{Z}_{ij}$ are Green's functions.

The tildes over the factors in Eqs.(1a) and (1b) imply Fourier transformation of the respective quantities. In this work, the Fourier trans-

formation is carried out in the bounded region interior to the shielding enclosure placed around the line segment representing the resonator. There we have the finite Fourier transformation:

$$\tilde{f}(k_n, \beta_m) = \int_{-rl/2}^{+rl/2} dz \int_{-a/2}^{+a/2} f(x,z)\ \exp(jk_n x)\ \exp(j\beta_m z)\, dx \tag{2}$$

where $k_{n,}\, \beta_m$ are the discrete transform variables defined by,

$k_n = (n - 1/2)\pi/a$ for E_z even, $- H_z$ odd (in x) modes

$k_n = n\pi/a$ for E_z odd , $- H_z$ even (in x) modes

$\beta_m = (m - 1/2)\pi/rl$ for E_z even, $-H_z$ odd (in z) modes

where a is the width of the waveguide enclosure, and rl is its length.

The analytical task then consists of assuming suitable coordinate forms for the current densities J_z and J_x, and Fourier-transforming these. Inner products are then formed in Eqs.(1a) and (1b), in accordance with the standard procedure of the method moments. This procedure gives rise to a set of homogeneous equations in the unknown coefficients assumed in J_z and J_x. The solution condition for the simultaneous equations leads to a determination of the resonance frequency of the SCPL resonator of the problem. The unknowns E_z and E_x can be eliminated by applying Galerkin's method in the spectral domain. The first

step is to expand the unknown J_z and J_x in terms of assumed basis functions J_{zi} and J_{xj} with unknown coefficients c_j and d_i.

$$\tilde{J}_z = \sum_{i=1}^{N1} d_i \tilde{J}_{zi}(k_n, \beta_m) \tag{3a}$$

$$\tilde{J}_x = \sum_{j=1}^{N2} c_j \tilde{J}_{xj}(k_n, \beta_m) \tag{3b}$$

The basis functions $\tilde{J}_{zi}$ and $\tilde{J}_{xj}$ must be chosen to be the Fourier transforms of space-domain functions $J_{zi}(x,z)$ and $J_{xj}(x,z)$ which are physically realistic, and which are zero except for the region $|x| < A/2$ and $|z| < l/2$. Substituting (3) into (1) yields the matrix equations,

$$\tilde{Z}_{zz} \sum_{i=1}^{N1} d_i \tilde{J}_{zi} + \tilde{Z}_{zx} \sum_{j=1}^{N2} c_j \tilde{J}_{xj} = \tilde{E}_z \tag{4a}$$

$$\tilde{Z}_{xz} \sum_{i=1}^{N1} d_i \tilde{J}_{zi} + \tilde{Z}_{xx} \sum_{j=1}^{N2} c_j \tilde{J}_{xj} = \tilde{E}_x \tag{4b}$$

Where $\tilde{J}_{zi} = \tilde{J}_{zi}(k_n, \beta_m)$, $\tilde{J}_{xj} = \tilde{J}_{xj}(k_n, \beta_m)$. Taking products of the resulting Eqs.(4a) and (4b) with the basis functions and for different values of i, j, yields the matrix equations:

$$\tilde{J}_{zk}\,\tilde{Z}_{zz}\sum_{i=1}^{N1} d_i\,\tilde{J}_{zi} + \tilde{J}_{zk}\,\tilde{Z}_{zx}\sum_{j=1}^{N2} c_j\,\tilde{J}_{xj} = \tilde{J}_{zk}\,\tilde{E}_z \tag{4c}$$

$$\tilde{J}_{xl}\,\tilde{Z}_{xz}\sum_{i=1}^{N1} d_i\,\tilde{J}_{zi} + \tilde{J}_{xl}\,\tilde{Z}_{xx}\sum_{j=1}^{N2} c_j\,\tilde{J}_{xj} = \tilde{J}_{xl}\,\tilde{E}_x \tag{4d}$$

Making summations of Eqs.(4c) and (4d) to complete the inner products, yields the matrix equations:

$$\sum_{m=-\infty}^{+\infty}\sum_{n=-\infty}^{+\infty}\tilde{J}_{zk}\tilde{Z}_{zz}\sum_{i=1}^{N1} d_i\tilde{J}_{zi}$$

$$+\sum_{m=-\infty}^{+\infty}\sum_{n=-\infty}^{+\infty}\tilde{J}_{zk}\tilde{Z}_{zx}\sum_{j=1}^{N2} c_j\,\tilde{J}_{xj} = \sum_{m}\sum_{n}\tilde{J}_{zk}\tilde{E}_z \tag{5a}$$

$$\sum_{m=-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} \tilde{J}_{xl} \tilde{Z}_{zx} \sum_{i=1}^{N1} d_i \tilde{J}_{zi}$$

$$+ \sum_{m=-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} \tilde{J}_{xl} \tilde{Z}_{xx} \sum_{j=1}^{N2} c_j \tilde{J}_{xj} = \sum_{m} \sum_{n} \tilde{J}_{xl} \tilde{E}_x \qquad (5b)$$

The right-hand sides of Eqs (5a). and (5b) are zero by virtue of Parseval's theorem, because the currents $J_{zi}(x)$, $J_{xj}(x)$ and the field components $E_z(x,d)$, $E_x(x,d)$ vanish in complementary regions of x. For example, when the inner product of $\tilde{J}_{zk} \tilde{E}_z$ on the right-hand side of Eqs.(5a) and (5b) is taken, $J_{zk}(x)$ is zero outside the strip, and $E_z(x)$ is zero on the strip. Therefore, the final boundary condition is satisfied. Equations (5a) and (5b) will be expressed in matrix form as follows:

$$\sum_{i=1}^{N1} d_i K_{ki}^{(1,1)} + \sum_{j=1}^{N2} c_j K_{ki}^{(1,2)} = 0 \qquad (5c)$$

$$\sum_{i=1}^{N1} d_i K_{li}^{(2,1)} + \sum_{j=1}^{N2} c_j K_{lj}^{(2,2)} = 0 \tag{5d}$$

Where

$$K_{ki}^{(1,1)}(\omega_o) = \sum_{m=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} \tilde{J}_{zk}(k_n, \beta_m) \tilde{Z}_{zz} \tilde{J}\, zi(k_n, \beta_m) \tag{6a}$$

$$K_{kj}^{(1,2)}(\omega_o) = \sum_{m=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} \tilde{J}_{zk}(k_n, \beta_m) \tilde{Z}_{zx} \tilde{J}_{xj}(k_n, \beta_m) \tag{6b}$$

$$K_{li}^{(2,1)}(\omega_o) = \sum_{m=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} \tilde{J}_{xl}(k_n, \beta_m) \tilde{Z}_{xz} \tilde{J}_{zi}(k_n, \beta_m) \tag{6c}$$

$$K_{lj}^{(2,2)}(\omega_o) = \sum_{m=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} \tilde{J}_{xl}(k_n, \beta_m) \tilde{Z}_{xx} \tilde{J}_{xj}(k_n, \beta_m) \tag{6d}$$

Where $\tilde{Z}_{pq} = \tilde{Z}_{pq}(k_n, \beta_m)$.

A homogeneous system of equations is thus obtained in terms of the unknown coefficients c_j, d_l . In order that c_j and d_l have nontrivial solutions, the determinant of the matrix must be zero, and hence the resonant frequency is determined for the resonator.

Equations (5) are now solved for the angular frequency ω_o by seeking the root of the resulting characteristic equation. The resonance frequency of the SCPL strip line resonator is derived from the obtained value of ω_o. The accuracy of the solution can be systematically improved by increasing the number of basis functions (N1, N2) and by solving larger size matrix equations. However, if the first few basis function are chosen so as to approximate the actual unknown current distribution reasonably well, the necessary size of the matrix can be held small for a given accuracy of the solution, resulting in numerical efficiency. Hence the choice of basis functions is important from the numerical point of view.

B. GREEN'S FUNCTION FOR THE MOMENT METHOD

Equations (1a) and (1b) may be expressed in the matrix form [Ref.2]:

$$\begin{bmatrix} \tilde{E}_z \\ \tilde{E}_x \end{bmatrix} = \begin{bmatrix} \tilde{Z}_{zz} & \tilde{Z}_{zx} \\ \tilde{Z}_{xz} & \tilde{Z}_{xx} \end{bmatrix} \begin{bmatrix} \tilde{J}_z \\ \tilde{J}_x \end{bmatrix} \tag{7}$$

where the transformed Green's functions have the values:

$$\tilde{Z}_{xz} = k_n \beta \frac{(RTH2 + RTH1)}{ZED} \tag{8a}$$

$$\tilde{Z}_{zz} = \frac{[(k_n^2 - \gamma_1^2)RTH2 + (k_n^2 - \gamma_2^2)RTH1]}{ZED} \tag{8b}$$

$$\tilde{Z}_{xx} = (k_n^2 - \gamma 1^2)RTH1 \frac{1}{ZED} \tag{8c}$$

$$\tilde{Z}_{zx} = \tilde{Z}_{xz}$$

$$ZED = (RTH1 + ER \times RTH2)\left(\frac{\gamma_1^2}{RTH1} + \frac{\gamma_2^2}{RTH2}\right)$$

Case1 $\quad \gamma_i^2 \geq 0$

$$RTH1 = \gamma_1 \tanh(\gamma_1 d)$$

$$RTH2 = \gamma_2 \tanh(\gamma_2 h)$$

case2 $\quad \gamma_i^2 < 0$

$$RTH1 = -\gamma_1 \tan(\gamma_1 h)$$

$$RTH2 = -\gamma_2 \tan(\gamma_2 h)$$

$$\gamma_i = \sqrt{k_n^2 + \beta_m^2 - \mu_o \varepsilon_i \omega^2}$$

Where i = 1,2 in the indicated layer. The quantities $\tilde{Z}_{zz}, \tilde{Z}_{zx}, \tilde{Z}_{xx}$ are actually the Fourier transforms of dyadic Green's function components.

C. CUT-OFF FREQUENCY IN TWO-DIELECTRIC LAYERED RECTANGULAR WAVEGUIDES

If a rectangular wave guide is partially filled with a solid dielectric arranged as indicated in Fig.1, this dielectric results in lowering the cut-off frequency and phase velocities of the modes as compared with that of a guide of the same dimensions wholly filled with air.

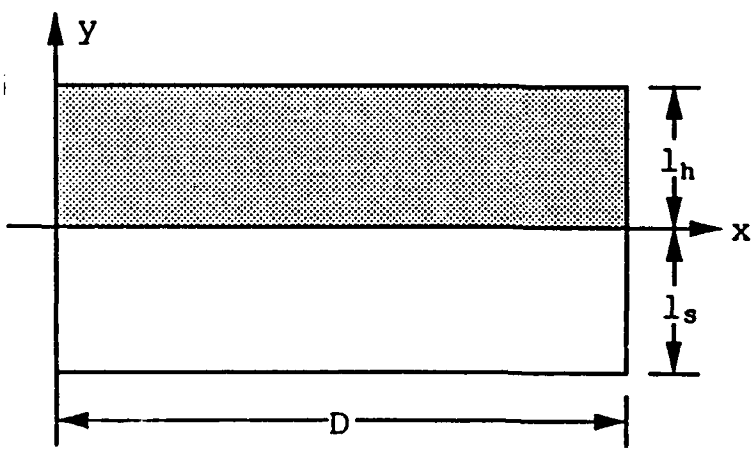


Figure 2. Cross sections of the waveguide

These effects have led to several important technical uses of layered-dielectric guides. The field in the guide is a linear combination of elemental waves, infinite in number, and termed normal modes. These modes are to be found by taking a z (axial) dependence of the form $\exp(jhz)$ for the six components of $\tilde{E}$ and $\tilde{H}$ and by constructing, through separation of variables, a wave which matches all boundary conditions. This procedure, as is illustrated below, yields a relation between h^2 and ω^2 termed the characteristic equation of such form that, for a given angular frequency ω, h^2 can take only a discrete, though infinite, set of values (the eigenvalues). To each eigenvalue corresponds a mode. When the eigenvalue h^2 is negative, h is imaginary, $\exp(jhz)$ is a damping factor and the mode is not propagated. When h^2 is positive, the mode is propagated. The transition between these two states occurs at a frequency termed the cut-off frequency of the mode, for which h^2 is equal to zero. The lowest of these modal cut-off frequencies is the fundamental cut-off frequency of the guide, below which no energy can be propagated in the guide. It follows from Maxwell's equations that the field components of

a mode can be expressed as a function of E_z and H_z, by the equations [Ref.11]:

$$(\frac{\omega^2}{c^2})\overline{E}_c = k^2\overline{E}_c = jh(\nabla E_z) + j\omega\mu(\overline{k}_x\nabla H_z) \qquad (9a)$$

$$((\frac{\omega^2}{c^2}) - h^2)\overline{H}_c = k^2\overline{H}_c = jh(\nabla H_z) - j\omega\varepsilon(\overline{k}_x\nabla E_z) \qquad (9b)$$

where the subscript c denotes a component in the plane of the cross section, $\overline{k}$ is a unit-vector directed along the z-axis, and $\overline{E}_c, \overline{E}_z, \overline{H}_c, \overline{H}_z$ determine the (x,y) dependent part of a component. Thus, typically

$$E_z = \overline{E}_z(x,y)\exp(j(\omega t + hz))$$

It also follows from Maxwell's equations that in each homogeneous medium, E_z and H_z satisfy Helmholtz's equation.

$$\nabla^2_{xy}A + k^2A = 0, \qquad A = (E_z, H_z)$$

Equations (9) and (10) enable expression of the boundary conditions on E_z and H_z. The conditions at a metallic wall are $E_z = \frac{\partial H_z}{\partial n} = 0$; the

conditions at a dielectric interface, in the particular case of Fig.2, are that the quantities:

$$\frac{jh}{k_i^2}\frac{\partial}{\partial x}E_z - \frac{j\omega\mu_i}{k_i^2}\frac{\partial}{\partial y}H_z \tag{10a}$$

and,

$$\frac{jh}{k_i^2}\frac{\partial}{\partial x}H_z + \frac{j\omega\varepsilon_i}{k_i^2}\frac{\partial}{\partial y}E_z, \qquad (i = h, s) \tag{10b}$$

be continuous at the interface. These considerations are applied to the guide depicted in Fig.2. Separation of variables in Eqs.(9a), (9b) and application of boundary conditions at the metallic walls yields

$$E_{zh} = A_h \sin(m\pi \frac{x}{d}) \sin(u_h \frac{(y - l_h)}{l_h}) \tag{11a}$$

$$E_{zs} = A_h \cos(m\pi \frac{x}{d}) \cos(u_s \frac{(y - l_s)}{l_s}) \tag{11b}$$

$$H_{zh} = B_h \cos(m\pi \frac{x}{d}) \cos(u_h \frac{(y - l_h)}{l_h}) \tag{11c}$$

$$H_{zs} = B_s \cos(m\pi \frac{x}{d}) \cos(u_h \frac{(y - l_s)}{l_s}) \tag{11d}$$

Where subscript h refers to the dielectric, and s to the air media, and m is integer (including zero). Let U_l be an auxiliary variable defined by

$$U_i = ((\frac{\omega^2}{c_i^2}) - h^2 - (\frac{m\pi}{d})^2 l_i)^{1/2}, \qquad i = h, s \tag{12}$$

Applying the boundary conditions at the dielectric interface through substituting Eqs.(11) in Eqs.(9) and (10) and setting y = 0 yields:

$$A_h \sin(U_h) + A_s \sin(U_s) = 0 \tag{13a}$$

$$B_h \cos(U_h) - B_s \cos(U_s) = 0 \tag{13b}$$

$$A_h \frac{hm\pi}{d} \frac{\sin(U_h)}{kh^2} + A_s \frac{hm\pi}{d} \sin \frac{(U_s)}{k_s^2}$$

$$+ B_h \frac{\omega U_h}{k_h^2} U_h \sin \frac{(U_h)}{l_h} + B_s \frac{\omega U_s}{k_s^2} U_s \frac{\sin(U_s)}{l_s} = 0 \tag{13c}$$

$$A_h \frac{\omega \varepsilon_h}{kh^2} \frac{U_h cos U_h}{l_h} - A_s \frac{\omega \varepsilon_s}{k_s^2} \frac{U_s cos U_s}{l_s} - B_h \frac{hm\pi}{d} \frac{cos U_h}{k_h^2}$$

$$+ B_s \frac{hm\pi}{d} \frac{cos U_s}{k_s^2} = 0 \tag{13d}$$

The determinant of the coefficients of this system of four homogeneous linear equations in A_h, A_s, B_h, B_s, must be zero in order that the field be nonvanishing. Formulating this condition yields the characteristic equation,

$$[\frac{K_m U_h tg U_h}{l_h k_h^2} + \frac{U_s tg U_s}{l_s k_s^2}][\frac{K_e U_h}{k_h^2 l_h tg U_h} + \frac{U_s}{k_s^2 l_s tg U_s}]$$

$$+ (\frac{hc_s m\pi}{\omega d})^2 \left(\frac{1}{k_s^2} - \frac{1}{k_h^2} \right)^2 = 0 \tag{14}$$

Wherein $K_e = \frac{\varepsilon_h}{\varepsilon_s}$ and $K_m = \frac{U_h}{U_s}$, and defining real auxiliary variables V_h, V_s by

$$V_h = [(\frac{\omega^2}{c_h})^2 - h^2 - (m\frac{\pi}{d})^2 l_h]^{1/2} \tag{15a}$$

$$V_s = [(m\frac{\pi}{d})^2 + h^2 - \omega^2 \frac{l_s}{c_s^2}]^{1/2} \tag{15b}$$

The factored characteristic equations for the cut-off frequency as obtained from equation (14) are:

$$(\frac{V_s th V_s}{k_s^2} l_s) - K_m V_h \frac{tg V_h}{l_h k_h^2} = 0 \tag{16}$$

$$(\frac{V_s}{k_{ss}^{2l} th V_s)} + K_e \frac{V_h}{k_h^2 l_h tg V_h} = 0 \tag{17}$$

where $tg = \tan$, $th = \tanh$

In Eq.(14), on determination of the cut-off frequency through imposition of $h = 0$, the second term disappears and Eqs.(10) can be factored into two simpler equations. V_s in Eqs.(16) and (17) cannot be real for $h = m = 0$. No single hyperbolic mode characterized by $m = 0$ can be propagated. By use of trial frequencies in Eqs.(16) and (17), we can find the cut-off frequency for $m = h = 0$. A fortran program for this calculation is shown in Appendix A. The cut-off frequency for the shielded coplanar line (SCPL) having dimensions shown in Table 1 was found to be 6.8 Ghz.

Table 1. CONFIGURATION DATA IN SCPL

dimension	data(mm)
D	22.86
l_h	2.54
l_s	7.62
ε_r	2.2

Dimensions are shown in Table 1 Symbols refer to Fig.2 as follows

D : width of shield

l_h : thickness of substrate

l_s : height of air

ε_r : dielectric constant

III. COMPUTER PROGRAM CONSTRUCTION

A. CURRENT DISTRIBUTIONS

In actual computations for the dominant mode, the strip current densities $\tilde{J}_{z1}$ and $\tilde{J}_{x1}$ have been chosen to have physically plausible distributions, as shown in Fig.3, for the continuous strip which has width of $2a$ and length of $2l_1$.

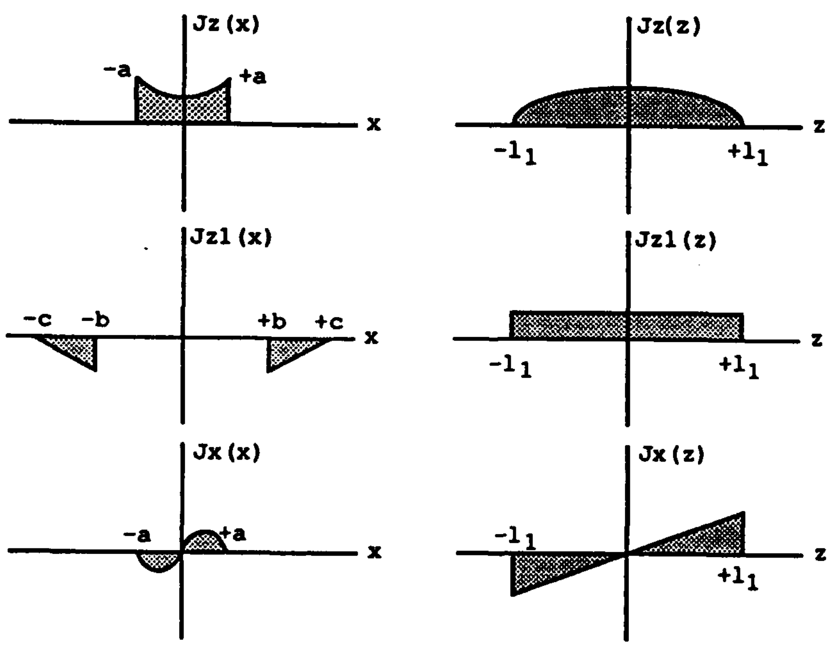


Figure 3. Form of assumed current distributions

Where $2a = A, 2b = B, 2c = C, 2l_1 = l$ (see Fig. 1).

The coordinate forms of the current distributions in Fig.3 are:

$$J_z(x) = \frac{1}{2a}(1 + |\frac{x}{a}|^3) \tag{18a}$$

$$J_{z1}(x) = \begin{cases} \dfrac{x-b}{2(c-b)} & x > 0 \\ \dfrac{-x+b}{2(c-b)} & x < 0 \end{cases} \tag{18b}$$

$$J_z(z) = \frac{1}{l_1}\cos\frac{(\pi z)}{2l_1} \tag{18c}$$

$$J_{z1}(z) = \frac{1}{2rl} \qquad |z| > \frac{rl}{2} \tag{18d}$$

$$J_x(x) = \frac{1}{a}\sin\frac{\pi x}{a} \tag{18e}$$

$$J_x(z) = \frac{z}{2l_1^{\,2}} \tag{18f}$$

Fourier transforms of these current densities are:

$$\tilde{J}_z(k_n) = \frac{2\sin(k_n a)}{k_n a} + \frac{3}{(k_n a)^2}[HWN] \tag{19a}$$

$$[HWN \equiv \cos(k_n a) - 2sin\frac{(k_n a)}{k_n a} + 2(1 - \cos(k_n a)\frac{1}{(k_n a)}^2]$$

$$\tilde{J}_{z1}(k_n) = (\frac{1}{b-c})(\frac{2}{k_n})[b\sin(k_n b) - csin(k_n b)]$$

$$+ (\frac{1}{b-c})[\frac{2}{k_n}(\cos(k_n b) - \cos(k_n c)] \tag{19b}$$

$$\tilde{J}_z(\beta_m) = \frac{4\pi\cos\beta_m l_1}{\pi^2 - (2\beta_m l_1)^2} \tag{20a}$$

$$\tilde{J}_{z1}(\beta_m) = \frac{\sin(\beta r l)}{\beta r l} \tag{20b}$$

$$\tilde{J}_x(k_n) = \frac{2\pi\sin(k_n a)}{(k_n a)^2 - \pi^2} \tag{20c}$$

$$\tilde{J}_x(\beta_m) = \frac{\cos(\beta_m l)}{\beta_m l_1} - \frac{\sin(\beta_m l_1)}{\beta_m l_1{}^2} \tag{20d}$$

And then, forming products of the x- and z- dependent factors,

$$\tilde{J}_z(k_n, \beta_m) = \tilde{J}_z(k_n).\tilde{J}_z(\beta_m) + \tilde{J}_{z1}(k_n).\tilde{J}_{z1}(\beta_m) \tag{21a}$$

$$\tilde{J}_x(k_n, \beta_m) = \tilde{J}_x(k_n).\tilde{J}_x(\beta_m) \tag{21b}$$

B. COMPUTATION PROCEDURE

1. Change of Transform Variable

In the present work, the strip line resonator was fully enclosed in a metal shield. This permitted the use of a finite Fourier transform rather than the integral transform which is typical of the infinite-line calculations.

The inner products can then be carried out as truncated summation, saving much computer time [Ref.9].

$$\sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} \rightarrow \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty}$$

In the products, the lower limits are $n = -\infty, m = -\infty$, unless symmetry holds, in which case the sum is:

$$\sum_{m=-\infty}^{\infty} = \sum_{m=0} + 2.\sum_{m=1}^{\infty}$$

The wave numbers used for the two summations are:

$$k_n = \frac{(n - \frac{1}{2})\pi}{a}, \quad x - direction$$

$$\beta_m = \frac{(m - \frac{1}{2})\pi}{rl}, \quad z - direction$$

As shown in Tables 2 and 3, a summation over n of 20 terms is enough, and for the summation over m, 1000 terms leads to convergence within 1 MHz error which is negligible in comparison with the resonant frequency.

Table 2. VARIATION OF NUMBER OF TERMS *N*

n	m	f_o Ghz
10	1000	3.318
20*	1000	3.3289
40	1000	3.3287

Table 3. VARIATION OF NUMBER OF TERMS M

n	m	f_o Ghz
20	100	3.23
20	500	3.317
20	1000*	3.328
20	1500	3.3287

C. CALCULATION OF FRINGING CAPACITANCE

Two equivalent representations of the electrical effects due to the fringing electromagnetic fields at the open ends of the strip transmission line section are available:

(a.) The stored energy in the fringing fields may be given an equivalent-circuit representation in the form of a small capacitance, ΔC, which is connected from the physical terminations of the line to ground.

(b.) The effect of the open end discontinuity may also be represented by assuming that the length of the open-ended strip is increased by the addition of length increment Δl to each end.

The two foregoing representations are related by the assumption that: $\Delta C = \Delta l C$, where C is the capacitance per unit length of the strip transmission line. Therefore in the present work it is necessary to determine

either ΔC or Δl, from the calculated resonance frequency of the strip resonator. Two methods are available for finding Δl or ΔC from resonance data: (1.) The half-wavelength resonator assumption, and (2.) the terminated transmission-line model for resonance of the open-ended strip.

1. HALF-WAVELENGTH RESONATOR ASSUMPTION

It is assumed that, at its fundamental resonance, the open-ended resonator has a length equal to one half wavelength of the propagating waves on the strip, apart from the perturbation due to the open ends. The wavelength can be found from a knowledge of the propagation constant $\beta(\omega_o)$, where ω_o is the angular frequency of resonance, i.e., $\lambda_g = \frac{2\pi}{\beta}$. Therefore, $\Delta l = \frac{1}{2}(\frac{\lambda_g}{2} - l)$ where l is the physical length of the resonator strip and λ_g is the guided wavelength in the strip. In view of the TEM model for the transmission line defined by L (henry/m) and C (farad/m), we have the relations:

$$Z_o = \sqrt{\frac{L}{C}} \tag{22}$$

$$\beta = \frac{\omega}{c}\sqrt{\varepsilon_{reff}} = \sqrt{LC}\,\omega_o \tag{23}$$

$$v = \frac{1}{\sqrt{LC}} = \frac{c}{\sqrt{\varepsilon_{reff}}} \tag{24}$$

where v is the wave velocity in SCPL, and c is the free- space velocity of light.

By using Eqs.(22) and (23), the capacitance C is defined

$$C = \frac{\beta}{Z_o \omega_o} \tag{25}$$

where C is capacitance/ meter of the transmission line, and Z_o is its characteristic impedance.

Therefore the fringing capacitance ΔC is

$$\Delta C = \Delta l C = \frac{1}{2}\left(\frac{\lambda_g}{2} - l\right)\frac{\beta}{Z_o \omega_o} \tag{26}$$

2. TERMINATED TRANSMISSION-LINE MODEL

It is assumed that the impedance seen looking from the center of the strip toward the open end has a zero value at the resonance of the strip resonator. This impedance can be calculated from the expression for the input impedance of a section of transmission line of length $\frac{l}{2}$, terminated by impedance Z_c (see Fig. 4):

$$Z_{in} = \frac{Z_c + jZ_o \tan(\beta \frac{l}{2})}{Z_o + jZ_c \tan(\beta \frac{l}{2})} Z_o \tag{27}$$

Where Z_o is the characteristic impedance of the line.

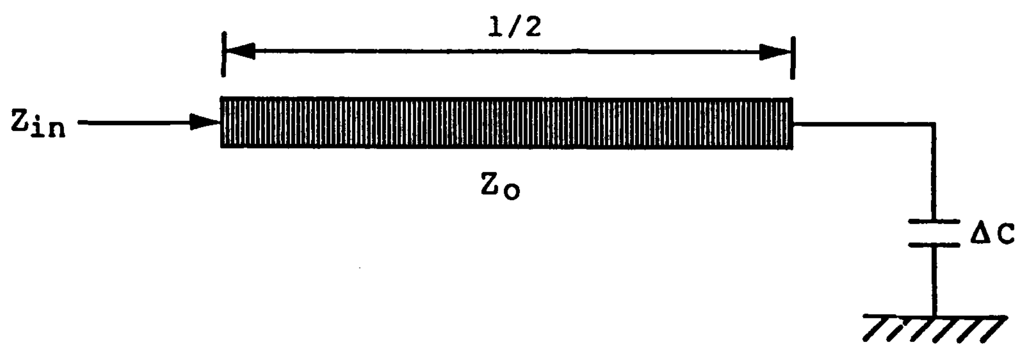


Figure 4. Terminated transmission line model

In the present case,

$$Z_c = \frac{1}{j\omega_o \Delta C} \tag{28}$$

$$\Delta C = \frac{1}{\omega_o Z_o} \cot(\beta \frac{l}{2}) \tag{29}$$

A plot of Eq (27), as a function of trial values of ΔC is shown in Fig.5. With the value of ω_o at resonance known, a CAD program may be used to determine the value of ΔC which leads to a zero of $| Z_{in} |$.

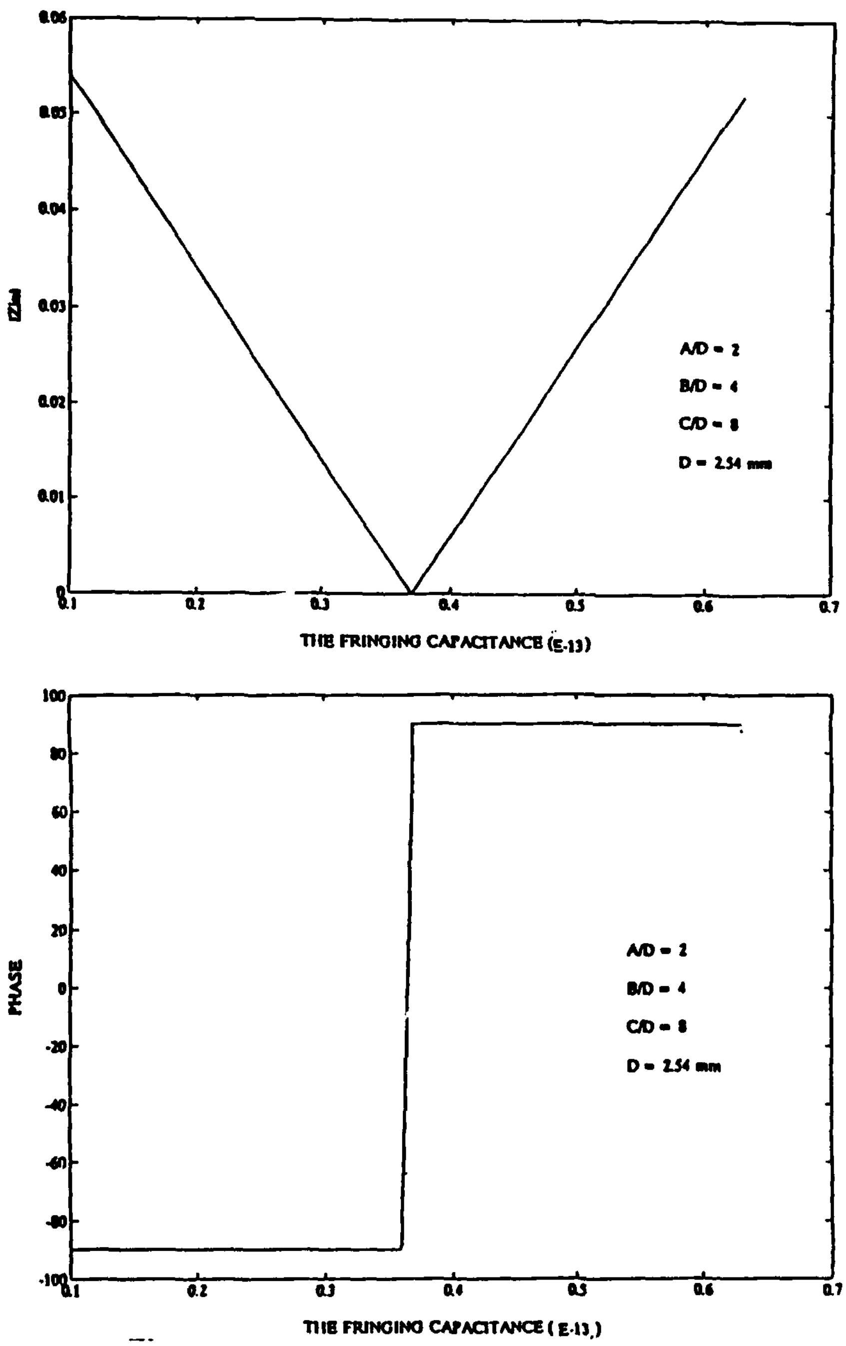


Figure 5. Magnitude and Phase of Z_{in} for terminated line - segment

IV. RESULT OF COMPUTATION

A. SCPL WITH VARYING DIELECTRIC CONSTANT

In Fig.6, A , B , and C were held fixed, and ε_r was increased from 2.2 to 4.0 and 10.0. The effect on ΔC, ε_{reff}, and Z_o and resonant frequency , f_o, was investigated. The value of ΔC is obtained by the two methods described above.

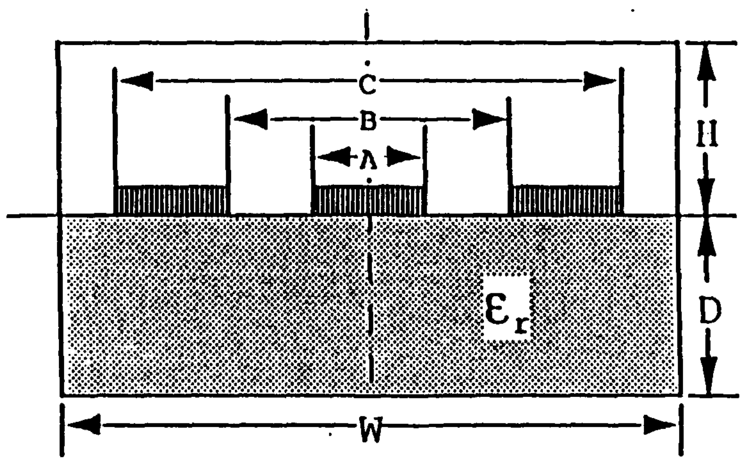


Figure 6. Cross section view of SCPL

Table 4. CONFIGURATION DATA IN SCPL

dimensions	data(mm)
A	5.08
B	10.16
C	20.32
W	22.86
H	7.62
D	2.54

Figure 7 shows that the anticipated effect is obtained. An increase of ε_r causes the fringing capacitance to increase and resonant frequency , f_o, to decrease. In Fig.8, the increasing ε_r causes Z_o to decrease, and ε_{reff} to increase. The calculation of ε_{reff} by the variational method or the alternate method of moments returns nearly the same value. Again, the results are consistent with the expected predictions.

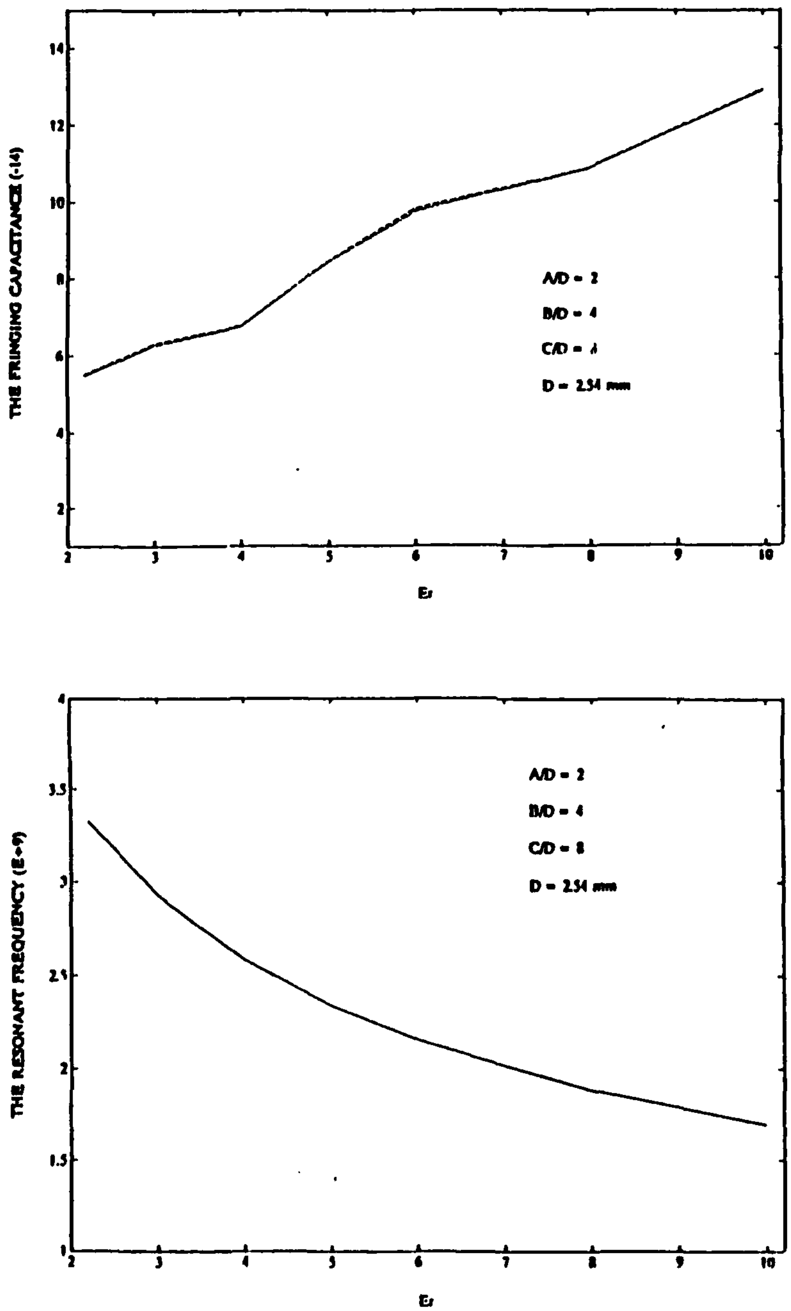
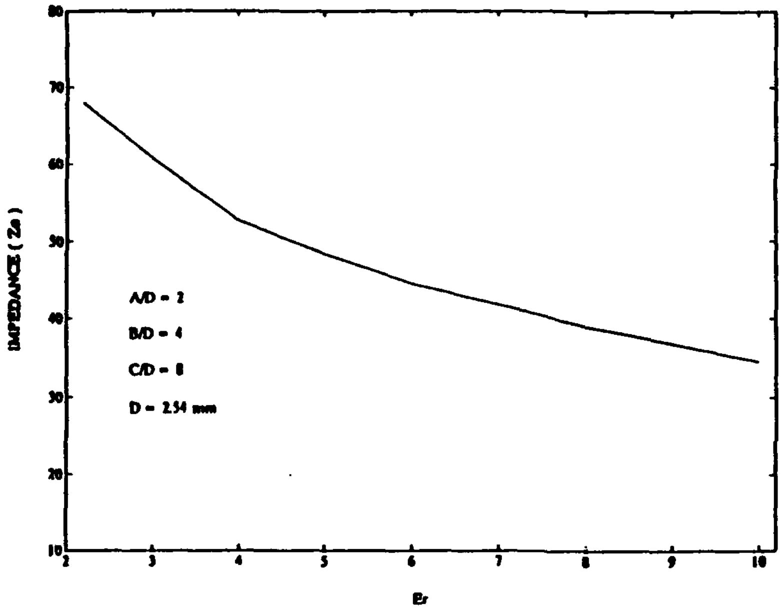


Figure 7. ΔC and f_o versus ε_r in SCPL



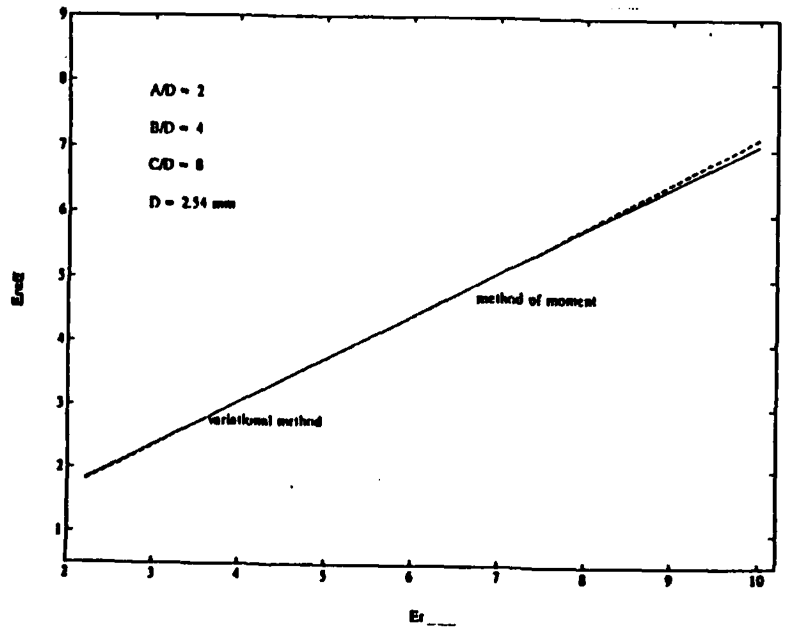


Figure 8. Z_o and ε_{reff} versus ε_r in SCPL

B. SCPL WITH VARYING CONDUCTOR SPACING

In Fig.6, the values A, D, H, GW remain fixed while the gaps between the center and ground conductors are increased (where GW is the ground-plane width). The effects on ΔC, ε_{reff}, and Z_o was analyzed. The increase of the gaps as shown in Fig.9 illustrates that resonant frequency, f_o, and ΔC remain relatively constant, independent of the SCPL gap width. Figure 10 shows a slight increase in impedance, Z_o, and in ε_{reff} with increase of gap width.

Table 5. CONFIGURATION DATA IN SCPL

dimension	data(mm)
D	2.54
H	7.62
A	2.54
GW	2.54
ε_r	2.2

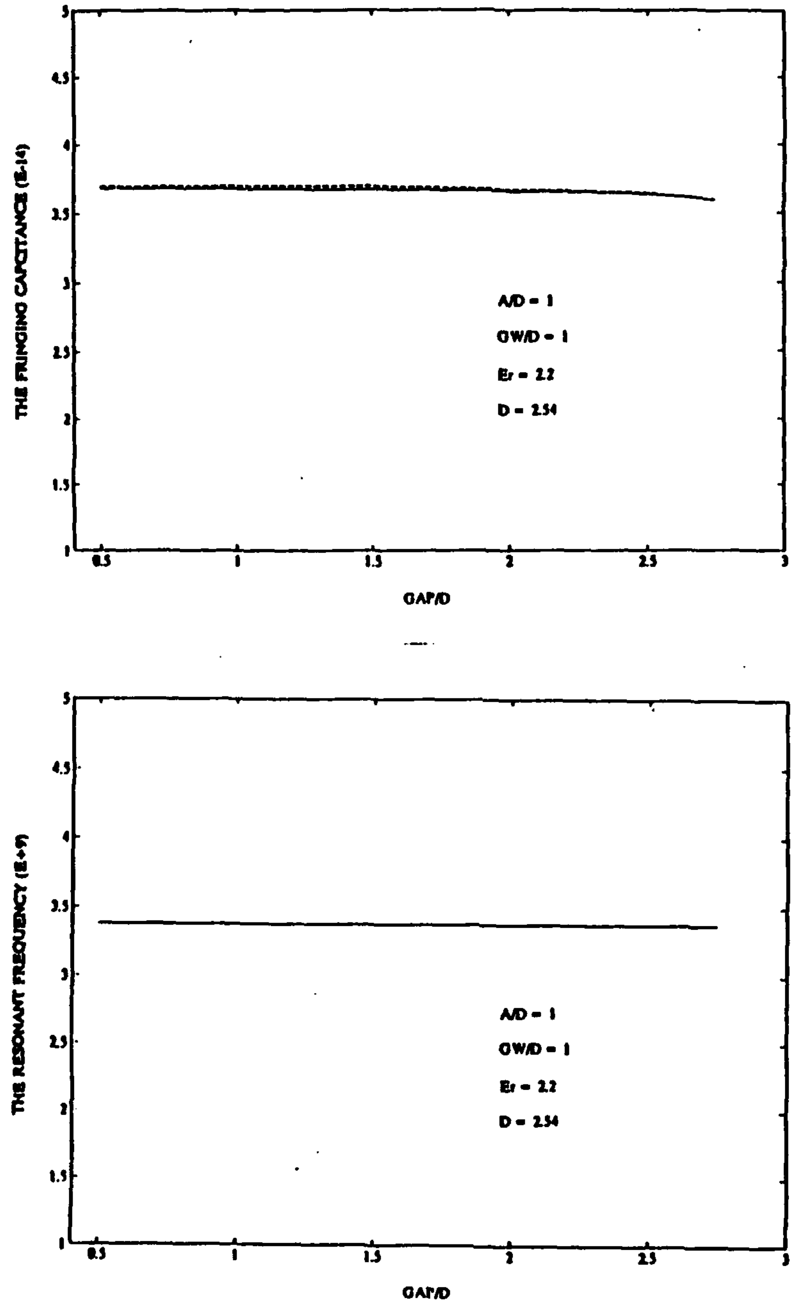


Figure 9. ΔC and f_o versus GAP/D in SCPL

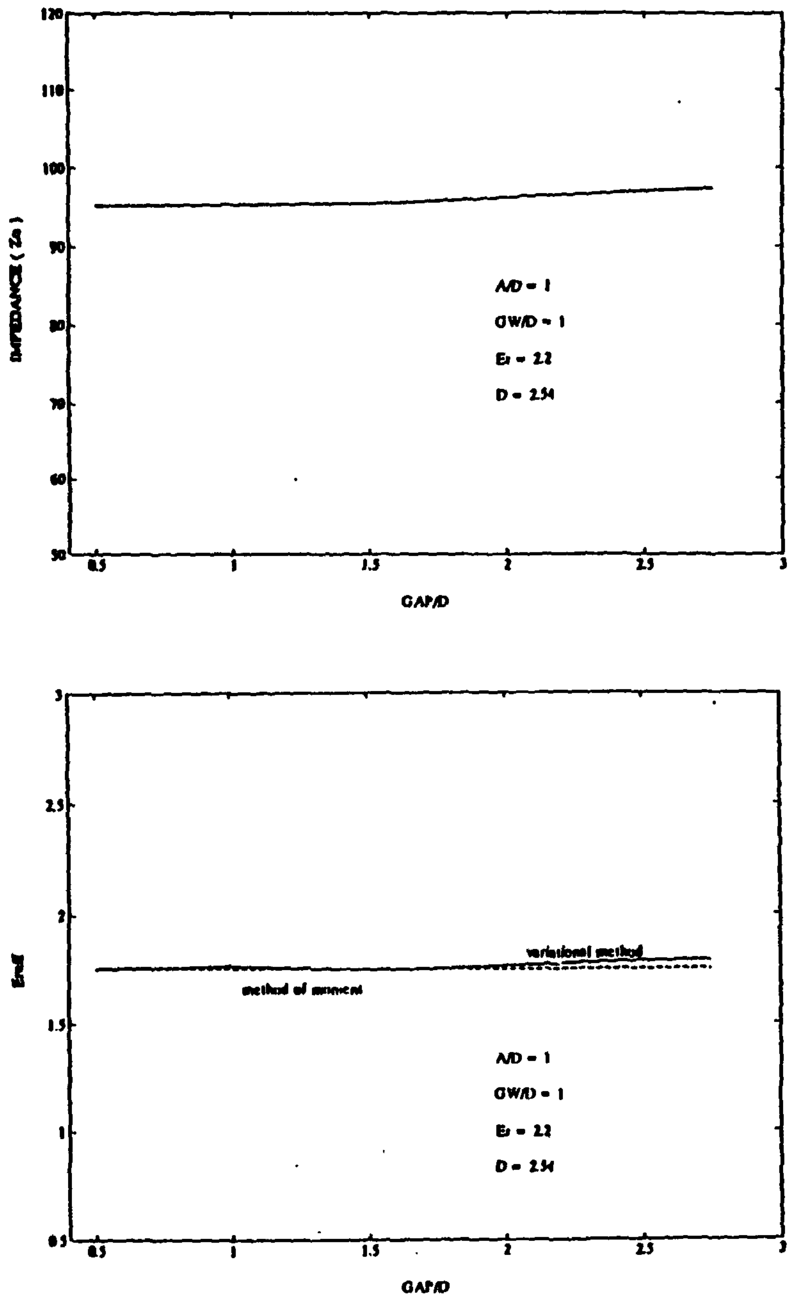


Figure 10. Z_o and ε_{reff} versus GAP/D in SCPL

C. SCPL WITH VARYING WIDTH OF THE CENTER CONDUCTOR

In Fig.6, the ground plane widths and values for H, D, W, ε_r are fixed. Only the width A of the center conductor is varied. This change necessarily changes the gap width also. The changing gap caused ΔC, f_o, ε_{reff}, and Z_o to change as will be discussed. The increasing width caused ΔC to decrease abruptly until A/D = 1. Above this ratio, ΔC increases slowly. Figure 11 illustrates that resonant frequency, f_o, increases until A/D is equal to 1. Above this ratio f_o remains relatively constant. In Fig.12, the increased width caused Z_o to decrease, and ε_{reff} remains relatively constant.

Table 6. CONFIGURATION DATA IN SCPL

dimensions	Data(mm)
W	22.86
GW	5.08
D	2.54
H	7.62
ε_r	2.2

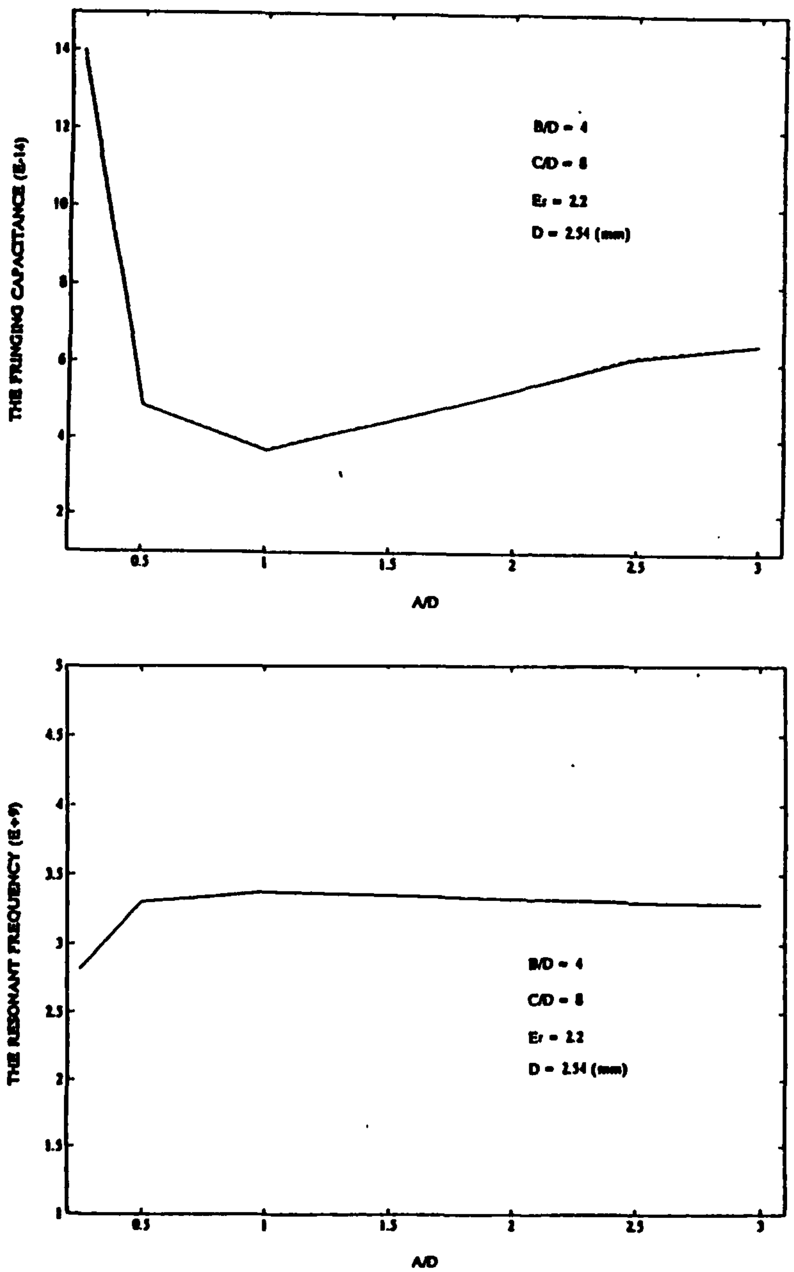
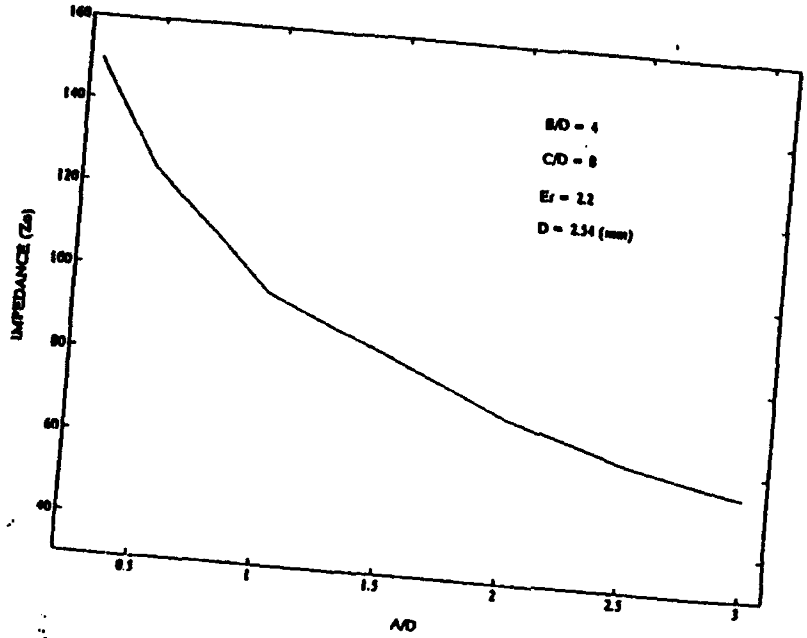


Figure 11. ΔC and f_o versus A/D in SCPL



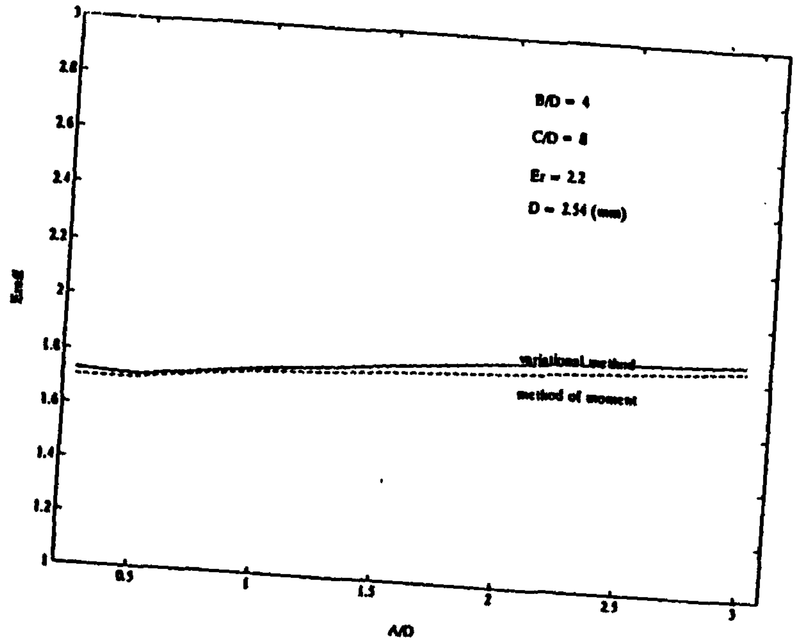


Figure 12. Z_o and ε_{reff} versus A/D in SCPL

D. SCPL WITH VARYING HEIGHT OF DIELECTRIC LAYER.

In Fig.6, A, B, C, and ε_r remain fixed while the dielectric layer height is increased from 1.27 to 6.35 mm . The effect on ΔC, ε_{reff}, and Z_o was investigated. Figure 13 shows that increasing D causes resonant frequency, f_o, to increase and the fringing capacitance, ΔC, to decrease slightly. Figure 14 shows that the increasing D causes Z_o to increase and ε_{reff} to decrease slightly.

Table 7. CONFIGURATION DATA IN SCPL

dimensions	Data(mm)
A	5.08
B	10.16
C	20.32
ε_r	2.2

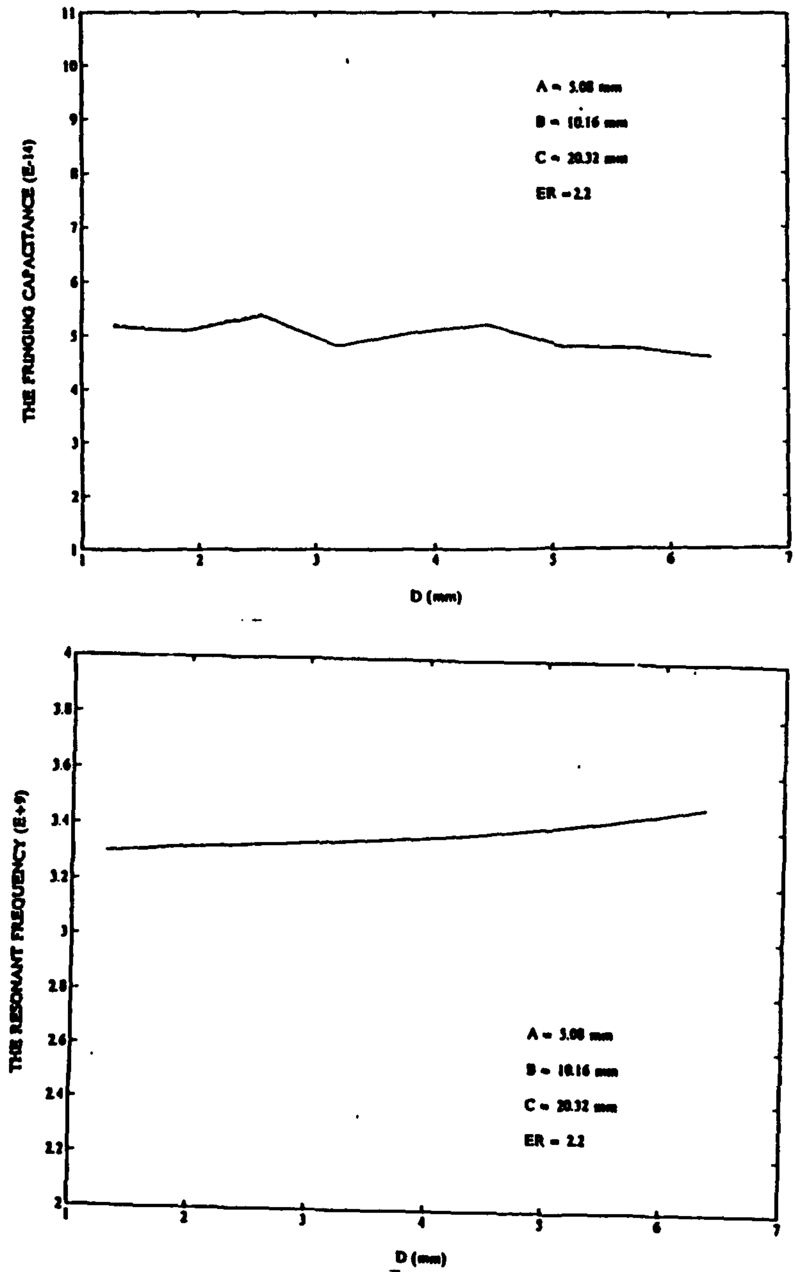


Figure 13. ΔC **and** f_o **versus D in SCPL**

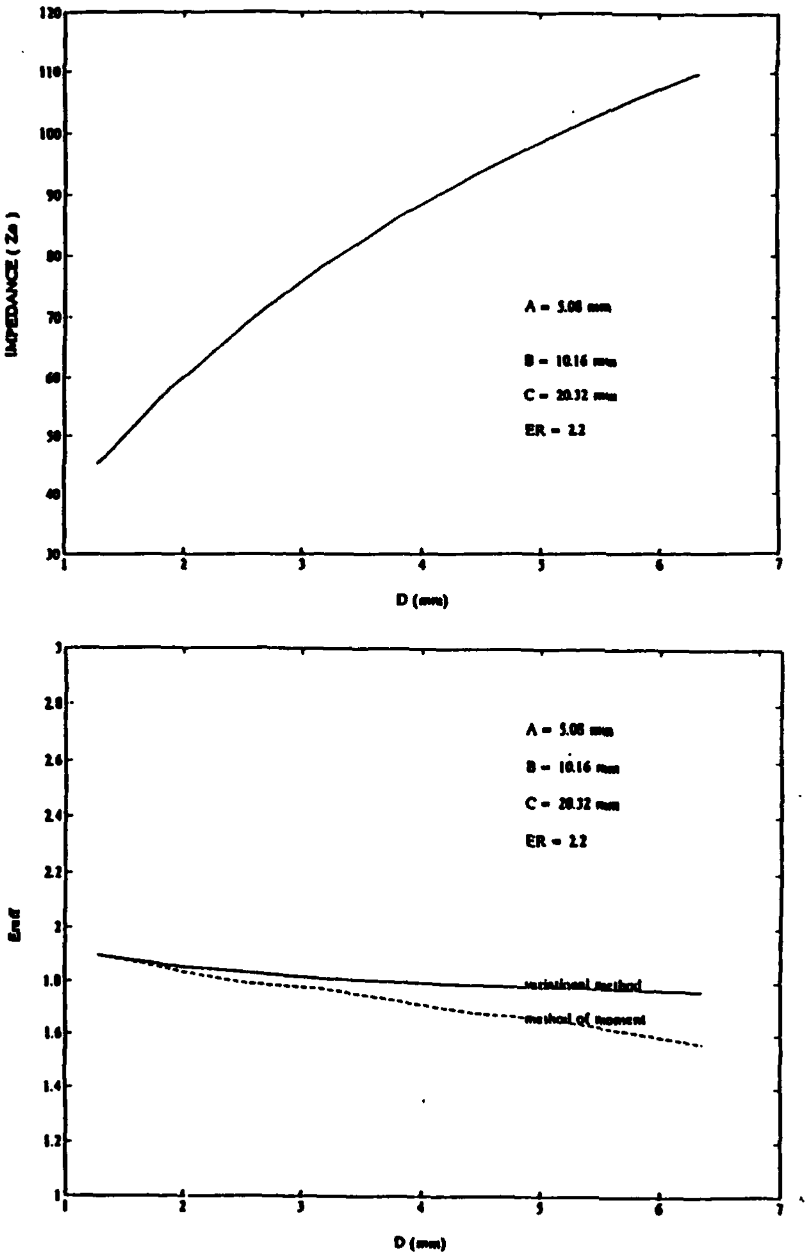


Figure 14. Z_o and ε_{reff} versus D in SCPL

V. CONCLUSION

This thesis follows the methods originated by Itoh for the analysis of the shielded micro strip resonator. A fully-shielded enclosure was employed, allowing the use of a finite Fourier transform in two coordinates. This change reduced the computation time significantly, while producing accuracy comparable with that obtained using the integral transform along the line axis. Also it should be mentioned that the operating frequency in the paper was taken to be below the cut-off frequency of SCPL, as verified by the thesis in Chapter II. The perturbed-resonator technique permitted the use of the strip current *density distributions* suggested by Itoh, and the ground strip current density distributions appropriate to the present work. These are known to give accurate results in Galerkin's method. The numerical values of open-end capacitances were obtained from two resonator models, and found to be in good agreement. In summary, values of the circuit model for the open-end discontinuity in shielded coplanar line has been investigated.

APPENDIX A. FORTRAN PROGRAMS

A. RESONANT FREQUENCY

```
      INTEGER I,N,M,COMPARE,COUNT
      REAL    W,A1,A2,L,D,H,ER,F,B,P,L1,L2
      REAL    KN,BETA,OMEGA,RL,Q
      REAL    PI,E0,MU0,C
      REAL    GAM1S,GAM2S
      REAL    JZIN,JZKN1,JZIN1,JZKN2,JZBE1,JZBE2,JZBE3,JZ,JZ1,JZ2,JZ3
      REAL    JXKN1,JXKN2,JXBE1,JXBE2,JX1,JX2,JX,JZKN3,JZBE4,JZ4
      REAL    CMP,LIMIT,OLDF,NEWF,LIMF
      DOUBLE PRECISION    DCMP
C
      LOGICAL LLOG
C
      COMPLEX*8 GAMA1,GAMA2,RTH1,RTH2
      COMPLEX*8 ZED
      COMPLEX*8 ZZZ,ZXX,ZXZ,ZZX
      COMPLEX*8 K11,K12,K22,J,SK11,SK12,SK22,DET
      CHARACTER*9    FNAME
      CHARACTER*12   SNAME
      Q=0.04
      OPEN(1,FILE='CXXXX')
      PRINT*, 'ENTER OUTPUT DATA FILE NAME FN'
      READ*, FNAME
      OPEN(2,FILE=FNAME)
      SNAME=FNAME//'PLT'
      OPEN(3,FILE=SNAME)
      WRITE(2,*) '    FILE NAME = ',FNAME
      WRITE(3,*) '    FILE NAME = ',SNAME
C
C     PARAMETER
      C=3.E8
      J=CMPLX(0.,1.)
      PI=4.*ATAN(1.)
      E0=1.E-9/(36.*PI)
      MU0=4.*PI*1.E-7
      LIMIT=1.E-2
      COMPARE=1
      COUNT=1
      LLOG=.TRUE.
C
      CALL READF(A1,A2,L1,L2,W,B,P,D,H,ER)
C
      WRITE(2,22) A1,A2,L1,L2,W,B,P,D,H,ER
 22   FORMAT(T3,'UNIT : METER'
     +     ,/,T3,'COPLANAR LINE WIDTH AND LENGTH : ',2(F9.7,2X)
     +     ,/,T3,'COPLANAR LINE   LENGTH L1,L2 : ',2(F9.7,2X)
     +     ,/,T3,'RESONATOR WIDTH                  : ',F9.5
     +     ,/,T3,'INSIDE  AND  OUTSIDE  WIDTH      : ',2(F9.7,2X)
     +     ,/,T3,'HEIGHTS OF LAYERS 1 AND 2        : ',2(F9.7,2X)
     +     ,/,T3,'PERMITTIVITY OF SUBSTRATE ER     : ',F9.5)
      PRINT*,'ENTER THE NUMBER OF OUTER SUM LOOP    N = '
      READ*,N
C     N=20
      WRITE(2,*) 'THE NUMBER OF OUTER SUM LOOP     N =',N
      WRITE(3,*) 'THE NUMBER OF OUTER SUM LOOP     N =',N
```

```
      PRINT*, 'ENTER THE NUMBER OF INNER SUM LOOP    M = '
      READ*, M
C     M=20, 40, 100, 500, 1000, 2000, 3000, 4000 ETC.
      WRITE(2,*) 'THE NUMBER OF INNER SUM LOOP      M =',M
      WRITE(3,*) 'THE NUMBER OF INNER SUM LOOP      M =',M
C
      L=L1+L2
      RL=5.*L
C
C
 1    PRINT*,'ENTER BOUNDARY FREQUENCY (GHZ)'
      READ*,F
      F=F*1.E9
C
 11   PRINT*,'FREQUENCY = ',F
      WRITE(6,*) 'FREQUENCY = ',F
C
      OMEGA=2.*PI*F
C
      K11=CMPLX(0.,0.)
      K12=CMPLX(0.,0.)
      K22=CMPLX(0.,0.)
      DO 20 I=1,N
         KN=(I-.5)*PI/W
C
         JZIN=COS(KN*A1)-2.*SIN(KN*A1)/(KN*A1)+2.*(1.-COS(KN*A1))/
     +     (KN*A1)**2
         JZKN1=2.*SIN(KN*A1)/(KN*A1)+3.*JZIN/(KN*A1)**2
         JZIN1=COS(KN*A2)-2.*SIN(KN*A2)/(KN*A2)+
     +       2.*(1.-COS(KN*A2))/(KN*A2)**2
         JZKN2=2*SIN(KN*A2)/(KN*A2)+3.*JZIN1/(KN*A2)**2
         JZKN3=Q*(1./(B-P)*(2./KN)*(B*SIN(KN*B)-P*SIN(KN*P))
     +          +1./(B-P)*(2./KN)*(COS(KN*B)-COS(KN*P)))
C
         JXKN1=-2.*PI*SIN(KN*A1)/(PI**2-(KN*A1)**2)
         JXKN2=-2.*PI*SIN(KN*A2)/(PI**2-(KN*A2)**2)
         SK11=CMPLX(0.,0.)
         SK12=CMPLX(0.,0.)
         SK22=CMPLX(0.,0.)
         DO 40 IM=1,M
            BETA=(REAL(IM)-.5)*PI/RL
C
C
C
            GAM1S=KN**2+BETA**2-E0*MU0*ER*OMEGA**2
C
            CALL GAMCT(GAMA1,GAM1S,RTH1,D)
C
C
            GAM2S=KN**2+BETA**2-E0*MU0*OMEGA**2
C
            CALL GAMCT(GAMA2,GAM2S,RTH2,H)
C
            ZED=(RTH1+ER*RTH2)*(GAMA1**2/RTH1+GAMA2**2/RTH2)
C
            ZXZ=KN*BETA*(RTH2+RTH1)/ZED
            ZZZ=((KN**2-GAMA1**2)*RTH2+(KN**2-GAMA2**2)*RTH1)/Z
            ZXX=((KN**2-GAMA1**2)*RTH2+(BETA**2-GAMA2**2)*RTH1)
```

```
C
              ZZX = ZXZ
              JZBE3 =SIN(BETA*L2)/(BETA*L2)
              JZBE4 =SIN(BETA*RL)/(BETA*RL)
C
C
C
              JZBE1=4.*PI*COS(BETA*(L1))/(PI**2-(2.*BETA*(L1))**2)
C
             JZBE2= 4.*PI*COS(BETA*(L2))/(PI**2-(2.*BETA*(L2))**2)
              JZ2=JZKN2*JZBE2
              JZ1=JZKN1*JZBE1
               JZ4=JZKN3*JZBE4
C
              JZ=0.5*(JZ1+JZ2)+JZ4
              JXBE1=COS(BETA*(L1))/(BETA*(L1))
     +        - SIN(BETA*(L1))/(BETA*(L1))**2
              JXBE2=COS(BETA*(L2))/(BETA*(L2))
     +           - SIN(BETA*(L2))/(BETA*(L2))**2
              JX1=JXKN1*JXBE1
              JX2=JXKN2*JXBE2
C
              JX=0.5*(JX1+JX2)
C
              SK11=SK11+JZ*ZZZ*JZ
              SK12=SK12+JZ*ZZX*JX
              SK22=SK22+JX*ZXX*JX
 40         CONTINUE
            K11=K11+SK11
            K12=K12+SK12


          K22=K22+SK22
20      CONTINUE
             PRINT*,'ZXX',ZXX,'ZZX',ZZX
C
          PRINT*,JX1,JX2,JX
          PRINT*, JZ1,JZ2,JZ3,JZ
C         PRINT*,ZZZ,ZZX,ZXX
C         PRINT*,'JX1',JX1,'JX2',JX2,'JXBE',JXBE
        PRINT*, 'CHECK K11 = ',K11
        WRITE(2,*) 'CHECK K11 = ',K11
        PRINT*, 'CHECK K12 = ',K12
        WRITE(2,*) 'CHECK K12 = ',K12
        PRINT*, 'CHECK K22 = ',K22
        WRITE(2,*) 'CHECK K22 = ',K22
        DET=K11*K22-K12*K12
        PRINT*, 'DET = ',DET
        WRITE(2,*) 'DETERMINENT = ',DET
C
C
        CALL FDSORT(F,DET,COUNT)
        COUNT=COUNT+1
```

```
C
C
      IF (LLOG) THEN
         PRINT*, 'MAKE SURE OPPOSITE SIGNS BETWEEN 1ST & 2ND DETS'
         PRINT*, 'IF SAME SIGNS THEN ENTER 9 , OR ENTER 0'
C
         READ*, IJK
         IF (IJK.EQ.9) THEN
            OLDF=F
            GOTO 1
         ENDIF
         LLOG=.FALSE.
      ENDIF
C
C
      CMP=REAL(DET)
      DCMP=DBLE(CMP)
      LIMF=ABS(F-OLDF)
C
      PRINT*, 'CHECK LIMIT FREQ ',LIMF
      WRITE(2,*) 'CHECK LIMIT FREQ = ',LIMF
C
      IF (ABS(DCMP).GT.LIMIT.AND. LIMF.GT.1.E4)  THEN
        CALL SLVAL(DCMP,OLDF,F,NEWF,COMPARE)
         OLDF=F
         F=NEWF
         GOTO 11
      ENDIF
C
      PRINT*, 'RESONANCE FREQUENCY = ',F
      WRITE(2,*) 'RESONANCE FREQUENCY = ',F
C
      CALL FOSORT(F,DET,100)

      STOP
      END


C****************************************************************
        SUBROUTINE READF(A1,A2,L1,L2,W,B,P,D,H,ER)
        REAL    A1,A2,L1,L2,W,B,P,D,H,ER
        INTEGER DDDD
9       PRINT*,'DATA INPUT : KEYBORD(ENTER "1"),FILE(ENTER "0")'
        READ*,DDDD
        IF(DDDD.EQ.0.OR.DDDD.EQ.0) THEN
             READ(1,*) A1,A2,L1,L2,W,B,P,D,H,ER
        ELSEIF(DDDD.EQ.1.OR.DDDD.EQ.1) THEN
             PRINT*,'ENTER THE 1,2 WIDTH  IN MM'
             READ*,A1,A2
             PRINT*,'ENTER THE  LENGTH OF MICRO STRIP LINE IN MM'
```

```
         READ*,L1,L2
         PRINT*,'ENTER THE WIDTH OF RESONATOR'
         READ*,W
         PRINT*, 'ENTER INSIDEWIDTH AND OUTSIDEWIDTH'
         READ*, B,P
         PRINT*,'ENTER THE HEIGHT OF LAYERS 1 AND 2'
         READ*,D,H
         PRINT*,'ENTER THE PERMITTIVITY ER OF SUBSTRATE  LAYER'
         READ*,ER
      ELSE
        PRINT*,'SELECT "1" OR "0"'
        GO TO  9
      ENDIF

      PRINT 32, A1,A2,L1,L2,W,B,P,D,H,ER
      WRITE(2,32) A1,A2,L1,L2,W,B,P,D,H,ER
      A2= A2*1.E-3
      A1= A1*1.E-3
      L1=L1*1.E-3
      L2=L2*1.E-3
      W = W*1.E-3
      B = B*1.E-3
      P = P*1.E-3
      D = D*1.E-3
      H = H*1.E-3
      ER=ER*1.0
32    FORMAT(T3,'UNIT:MILIMETER'
     + ,/,T3,'MICRO STRIP LINE 1,2 WIDTH          :',2(F9.7,3X)
     + ,/,T3,'MICRO STRIP LINE WIDTH L1,L2        :',2(F8.3,3X)
     + ,/,T3,'RESONATOR WIDTH                     :',F9.5
     + ,/,T3,'INSIDE  AND OUTSIDE  WIDTH          :',2(F9.5,3X)
     + ,/,T3,'HEIGHT OF LAYERS 1,2                :',2(F9.5,3X)
     + ,/,T3,'PERMITTIVITY OF SUBSTRATE  ER      :',F9.3)
      RETURN
      END
C*****************************************************************
      SUBROUTINE GAMCT(GAMA,GAMS,CT,LENGTH)
      COMPLEX*8 GAMA,CT
      REAL  GAMS,LENGTH,GAMR,CTR
      GAMR = SQRT(ABS(GAMS))
      IF(GAMS.LT.0.  ) THEN

          GAMA = CMPLX(0.,GAMR)
          CTR =-GAMR*TAN(GAMR*LENGTH)
          CT = CMPLX(CTR,0.)
       ELSE
          GAMA = CMPLX(GAMR,0.)
          CTR = GAMR*TANH(GAMR*LENGTH)
          CT = CMPLX(CTR,0.)

       ENDIF
       RETURN
       END
```

```
C*****************************************************************************
      SUBROUTINE SLVAL(FUN,OXV,XV,NEWXV,SN)
      INTEGER SN
      REAL        OXV,XV,NEWXV,POSXV,NEGXV
      DOUBLE PRECISION    FUN
      IF(SN.EQ.1) THEN
       IF(FUN.GT.0) THEN
            POSXV = XV
            NEGXV = OXV
         ELSE
            NEGXV = XV
            POSXV = OXV
        ENDIF
        NEWXV = (POSXV + NEGXV)/2
        SN = SN +2
     ELSE
       IF(FUN.GT.0) THEN
          POSXV = XV
       ELSE


         NEGXV = XV
       ENDIF
       NEWXV = (POSXV +NEGXV)/2
     ENDIF
     RETURN
     END
C*****************************************************************************
      SUBROUTINE  FDSORT(FQY,SDET,ICOUNT)
      INTEGER ICOUNT,IC
      REAL    FREQY(100),FQY,RTEMP,MAGDET,TFQY
      COMPLEX*8   DETMT(100),SDET,CTEMP,TSDET
      TFQY = FQY
      TSDET = SDET
     IF(ICOUNT.EQ.1) THEN
       FREQY(1) = TFQY
       DETMT(1) = TSDET
     ELSEIF(ICOUNT.NE.100) THEN
       DO 10 I=1,ICOUNT-1

         RTEMP = FREQY(I)
         CTEMP = DETMT(I)
         IF (RTEMP.GT.TFQY) THEN
              FREQY(I) = TFQY
              TFQY = RTEMP
              DETMT(I) = TSDET
              TSDET = CTEMP
         ENDIF
10     CONTINUE
       FREQY(ICOUNT) = TFQY
       DETMT(ICOUNT) = TSDET
       IC = ICOUNT
     ELSEIF(ICOUNT.EQ.100) THEN
       WRITE(3,110)
110    FORMAT(//,T5,'FREQUENCY ',T20,'MAG OF DET ',T35,'DETERMINENT')
       DO 20 J  =1,IC
         MAGDET = SQRT(REAL(DETMT(J))**2+AIMAG(DETMT(J))**2)
         WRITE(3,210) FREQY(J),MAGDET,DETMT(J)
210      FORMAT(T5,F15.2,T20,F15.3,T35,2(E15.8))
20     CONTINUE
     ENDIF
     RETURN
     END
```

B. THE FRINGING CAPACITANCE

```
* THIS IS THE PROGRAM OF CALCULATING  THE CAPACITANCE
       REAL      ZCAP,Z0,BETA,L,ZIN,F,TH,PI
       REAL      A,U,T1
       INTEGER T
       CHARACTER*8 FNAME
       PI = 3.14
       PRINT*,'ENTER OUTPUT FILENAME'
       READ*,FNAME
       OPEN(2,FILE=FNAME)
       WRITE(2,*) ' FILE NAME =', FNAME
* THE PROGRAM OF FINDING TAN(BETA*L)
       PRINT*,'ENTER BETA'
       READ*,BETA
       PRINT*,'ENTER L (MM)'
       READ*,L
       L=L/1000.
       PRINT*,'ENTERRESONANT FREQUENCY FO(GHZ)'
       READ*,F
       F=F*1.E9
       PRINT*,'ENTER Z0'
       READ*,Z0
       TH = TAN(BETA*L)
       PRINT*,'BETA',BETA
       PRINT*,'F',F
       PRINT*,'L',L
       PRINT*,'Z0',Z0
* FINDING THE ZIN VALUE
        WRITE(2,110)
110     FORMAT(//,T5,' TH',T20,'ZIN',T35,'CAPACITANCE')
        DO 100 T = 10 ,200
        T1=T*1.E-15
        ZCAP=1./(T1*PI*2*F)
        ZIN = (-ZCAP+Z0*TH)/(Z0+(ZCAP*TH))
        PRINT*,'CF',T1
        PRINT*, 'ZIN',ZIN
        WRITE(2,210) TH, ZIN,T1
210     FORMAT(T5,F15.2,T20,F15.3,T35,2(E15.8))
100     CONTINUE
         END
```

C. PROPAGATION CONSTANT

```
      INTEGER   MMAX,NMAX
      REAL   W,A,D,T,H,PI,ER,FG,OMG,E0,KN,G1S2,B0,B00,DBTA,FLAG,AA2,CC,WW
      REAL   G1,CT01,CT03,GT1,GT3,G2S2,G2,CT02,KE1,KH1,JZ1
      REAL   B,OUT,BIN
      DIMENSION  B(60),OUT(30)
      COMPLEX  GM1,GM3,CT1,CT3,GM2,CT2,DENE,ZE,KE,DENH,ZH,KH
      COMPLEX  TRMN,SUMUP,JZ11
C     READ*,FNAME
      PI=4.*ATAN(1.)
      FLAG=0
      MMAX=30
      NMAX=30
      PRINT*,'ENTER  FREQ.,GHZ.'
      READ*, FG
C     PRINT*,'ENTER  SHLD.  WDTH(MM) W'
C     READ*, AA2
      AA2=22.85
      A=AA2/2000
C     PRINT*,'ENTER  SHLD.  HT.(MM) L'
C     READ*,CC
      CC1=10.16
      CC=CC1/1000
       D1=1.E-60
       D=D1/1000
      PRINT*,'ENTER SCPL THICKNESS(MM) D'
      READ*,T
C     T1=2.54
      T=T/1000
      H=CC-(T)
      PRINT*,'ENTER THE SCPL DIELEC. CONST. '
      READ*,ER
C      ER= 4.0
      PRINT*,'ENTER THE LINEWIDTH(MM) 2*W'
      READ*,WW
      W=WW/2000
      OMG=FG*PI*2E9
      E0=8.85E-12
      B0=(OMG/3E8)*SQRT((ER+1)/2)
C     B0=(OMG/3E8)*SQRT(EREFF)
C     B0=(OMG/3E8)*SQRT(ER)
      B00=0.20
      DBTA=0.10
      GO TO 4
    5 B0=BIN
      B00=0.85
      DBTA=0.0070
    4 DO  2  M=1,MMAX
      SUMUP=0
      B(M)=B0*(B00+M*DBTA)
      DO  1  N=1,NMAX
      KN=(N-0.5)*PI/A
```

```
      G1S2=KN**2+B(M)**2-OMG**2/9E16
      G1=SQRT(ABS(G1S2))
      IF(G1S2.LT.0) THEN
         GM1=CMPLX(0.,G1)
         GM3=GM1
         CT01=-1./TAN(G1*H)
        CT03=-1./TAN(G1*D)
         CT1=CMPLX(0.,CT01)
        CT3=CMPLX(0.,CT03)
      ELSE
         GM1=CMPLX(G1,0.)
         GT1=1./TANH(G1*H)
         GM3=GM1
         GT3=1./TANH(G1*D)
         CT1=CMPLX(GT1,0.)
         CT3=CMPLX(GT3,0.)
      ENDIF
C
      G2S2=KN**2+B(M)**2-ER*OMG**2/9.E16
      G2=SQRT(ABS(G2S2))
      IF (G2S2.LT.0) THEN


        GM2=CMPLX(0.,G2)
        CT02=-1./TAN(G2*T)
        CT2=CMPLX(0.,CT02)
      ELSE
        GM2=CMPLX(G2,0.)
        GT2=1./TANH(G2*T)
        CT2=CMPLX(GT2,0.)
      ENDIF
C
      DENE=CT2*CT3+CT1*CT3*(GM2/GM1)/ER+CT1*CT2+(GM3/GM2)*ER
      ZE=(GM2*CT3/ER+GM3*CT2)/DENE
      KE1=-1./(OMG*E0)
      KE=CMPLX(0.,KE1)
      ZE=KE*ZE
      DENH=GM1*CT1*GM2*CT2+GM1*CT1*GM3*CT3+GM2*CT2*GM3*CT3+GM2**2
      ZH=(GM2*CT2+GM3*CT3)/DENH
      KH1=OMG*PI*4E-7
      KH=CMPLX(0.,KH1)
      ZH=ZH*KH
      JZ1=COS(KN*W)-2*SIN(KN*W)/(KN*W)+2*(1-COS(KN*W))/(KN*W)**2
      JZ1=2.*SIN(KN*W)/(KN*W)+3./((KN*W)**2)*JZ1
      JZ11=CMPLX(JZ1,0.)
      TRMN=-(B(M)**2*ZE+KN**2*ZH)/(B(M)**2+KN**2)*JZ11**2
      SUMUP=TRMN+SUMUP
    1 CONTINUE
C
      OUT(M)=CABS(SUMUP)
    2 CONTINUE
C
      DO  3  M=2,(MMAX-1)
      IF ((OUT(M).LT.OUT(M-1)).AND.(OUT(M).LT.OUT(M+1))) THEN
        BIN=B(M)
      ELSE
      ENDIF
    3 CONTINUE
C
      IF (FLAG.EQ.0) THEN
        FLAG=1.
        GO TO 5
      ELSE
        PRINT*,'SCPL BETA = ',BIN
        WRITE(2,*)  'SCPL BETA = ',BIN
      ENDIF
      STOP
      END
```

D. CUT - OFF FREQUENCY

```
* THIS IS THE PROGRAM CALCULATING FOR CUT OFF FREQUENCY OF SCPL
       REAL L,LS,LH,D,ER,PI,M,FG,KS,KH ,VH,A,TGH,C,F3,F4,F5
       COMPLEX VS,THS,F1,F2,CTEMP1,CTEMP2
       PRINT*,'VON BLADEL HIGGINGS CUTOFFS FOR LOADED GUIDE'
C      PRINT*,'ENTER UUTFILE'
C      READ*,FL
       OPEN(UNIT=2,FILE ='FL')
       PI = 4*ATAN(1.)
       D = .02286
       L = .01016
       M = 0
       C=3E8
* FRACTIONAL HT.. OF DIELEC. LAYER:
       LS = 5.77/1000.
* DIEL. LAYER ER:
       ER = 10.0
       PRINT*,'GUIDE WDTH AND HT',D,L
       PRINT*,'DIEL LAYER HT AND ER',LH,ER
       LH=-3.37/1000.
       PRINT*,'FREQ.(GHZ) ','MAG.(F1)  ','MAG.(F2)'
       WRITE(2,*) 'LS=',LS,'LH=',LH,'ER=', ER
       DO 99, A=1,10000
       FG= 3.0+.1*A
       KS = 20*PI*FG/3
       KH= KS*SQRT(ER)
       CTEMP1= (KH**2-(M*PI/D)**2*LH )
       VH=CSQRT(CTEMP1)
       CTEMP2= ( (M*PI/D)**2 - KS**2*LS )
       VS= CSQRT(CTEMP2)
C      TGH=(CEXP(VH)-CEXP(-VH))/(CEXP(VH)+CEXP(-VH))
       TGH =SIN(VH)/COS(VH)
C       TGH =C(GH)
       THS = ( CEXP(VS)-CEXP(-VS))/(CEXP(VS)+CEXP(-VS))
       F1 = ((VS*THS)/(LS*KS**2))-((VH*TGH)/(LH*KH**2))
       F2 = VS/(KS**2*LS*THS) + ((VH*ER)/(KH**2*LH*TGH))
       F3=SQRT(ER)*(2*PI*FG)/C*TAN(SQRT(ER)*2*PI*FG/C)
       F4=SQRT(ER)*(2*PI*FG)/C*TAN(2*PI*FG/C)
       F5=F3+F4
       T=F1*F2
55      FORMAT(1X,3F10.5)
       PRINT 55, FG, CABS(F1),CABS(F2)
       WRITE(2,*)  FG, CABS(F1),CABS(F2)
        WRITE(2,*) 'F5=',F5
99     CONTINUE
       END
```

APPENDIX B. IMPEDANCE AND DIELECTRIC CONSTANT BY VARIATIONAL METHOD

A method for computing impedance (Z_o) and effective dielectric constant ε_{reff} of the shielded coplanar line (SCPL) is based on the application of the Fourier transform and variational techniques[Ref.10].

The computation was found to be insensitive to the form assumed for the charge distribution on the top ground planes, hence a uniform (negative) charge distribution was assumed.

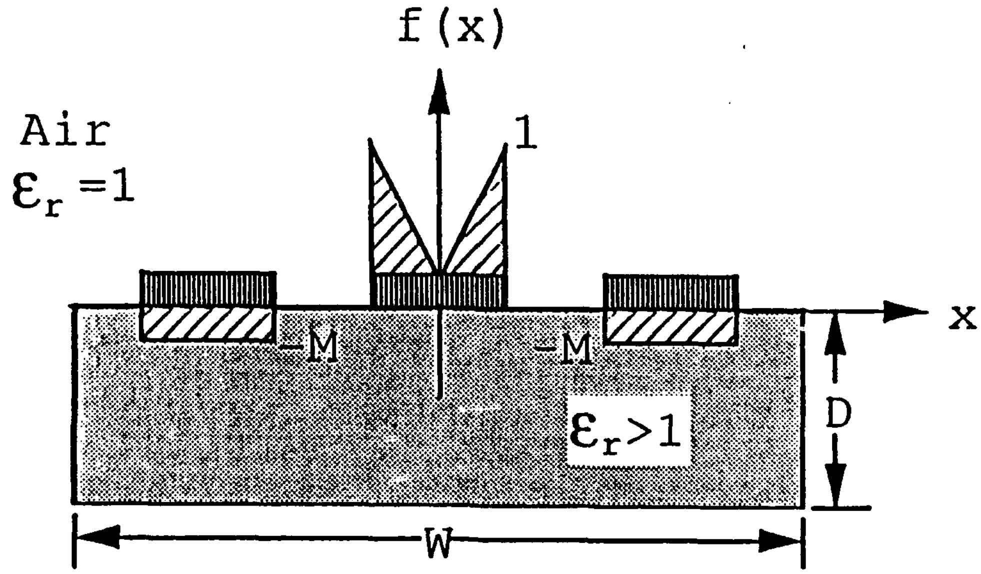


Figure 15. Assumed line-charge density function of SCPL

In case of Fig.15, the assumed elementary charge distribution is:

$$f(x) = \begin{cases} |x| & \frac{-A}{2} \le x \le \frac{A}{2} \\ -M & \frac{B}{2} \le |x| \le \frac{P}{2} \\ 0 & \text{otherwise} \end{cases} \qquad (a1)$$

Where p = the width C in Fig.1.

The Fourier transform of f(x) is :

$$\tilde{f}(\beta) = Q\left[\frac{2sin(\beta \frac{A}{2})}{\beta \frac{A}{2}} - \{ \sin \frac{(\beta \frac{A}{4})}{\beta \frac{A}{4}} \}^2 \right.$$

$$\left. + \frac{2M}{\beta} \{ \sin(\beta \frac{B}{2}) - \sin(\beta \frac{P}{2}) \right] \qquad (a2)$$

and

$$Q = \frac{A}{2} + M(B - P) \qquad (a3)$$

Using the computation given in Ref.10, the characteristic impedance is found to be,

$$Z_o = \frac{1}{v\sqrt{CC_o}} \qquad (a4)$$

Where C and C_o are the variational values of line capacitance with the given dielectric and with air dielectric, respectively.

$$\varepsilon_{reff} = \frac{C}{C_o} \qquad (a5)$$

where $v = 3 \times 10^8 m/s$, and C_o is calculated by setting $\varepsilon_r = 1$.

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